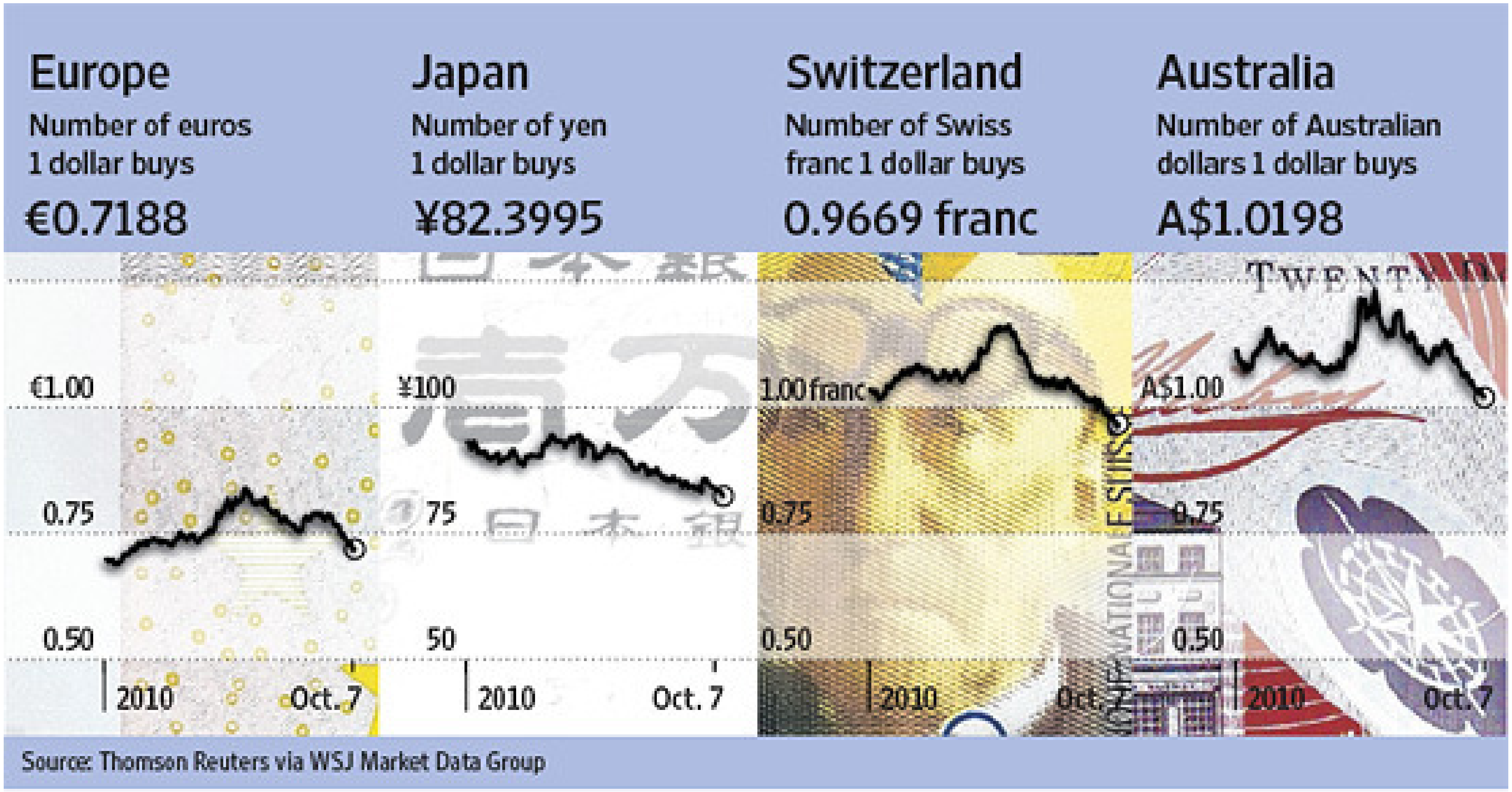
Markets Update

“Dow Tops 11000” WSJ, Oct. 8

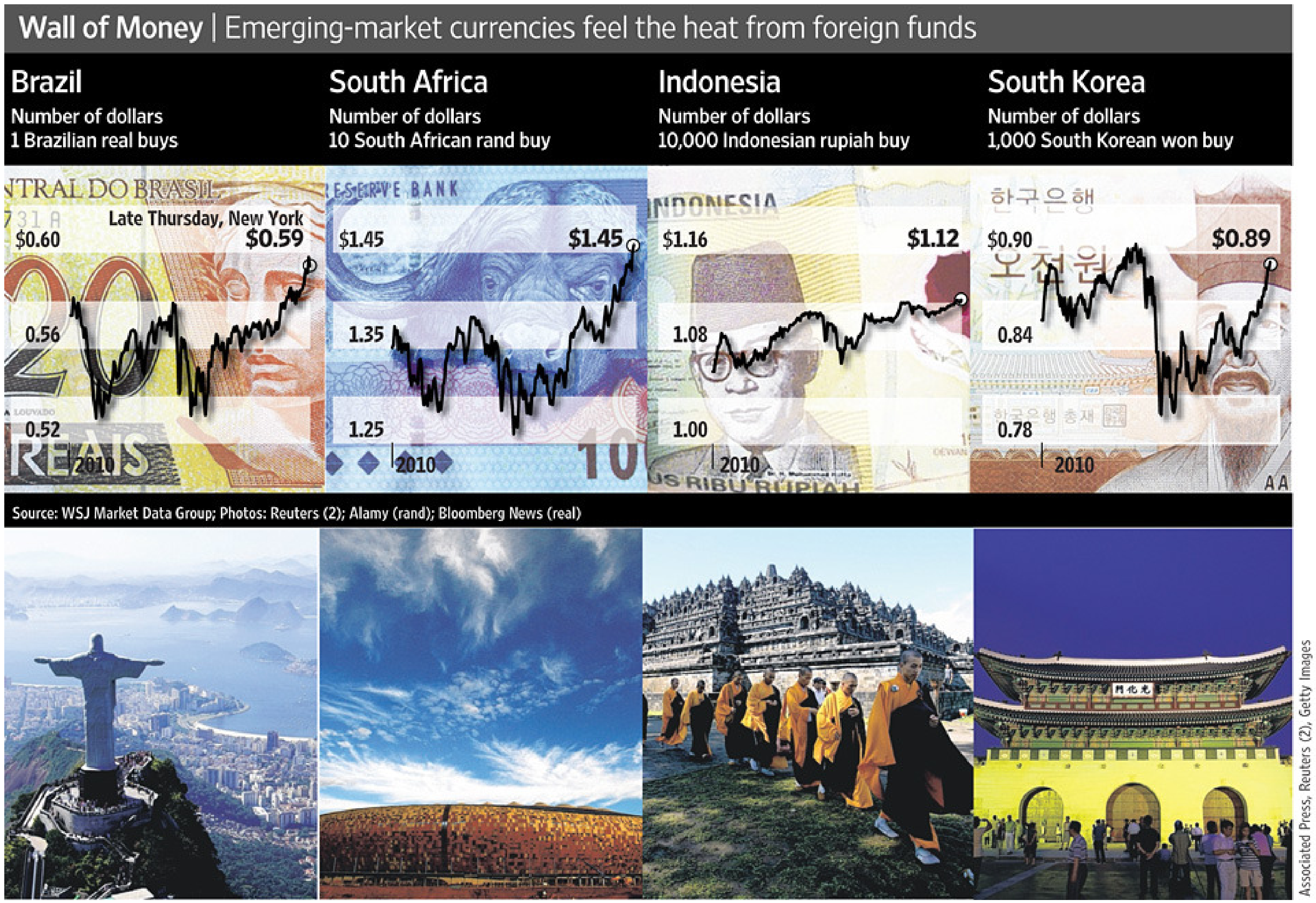
“Bond Investors Await Fed's Move “ WSJ, Oct. 8

Two-Year, Five-Year Yields Scrape New Lows on Policy Bets

Markets Update



Markets Update



Chapter 6

Efficient Diversification

(Cont'd)

McGraw-Hill/Irwin

Covariance and correlation

- The problem with covariance
  - Covariance does not tell us the intensity of the comovement of the stock returns, only the direction.
  - We can standardize the covariance however and calculate the correlation coefficient which will tell us not only the direction but provides a scale to estimate the degree to which the stocks move together.

The correlation coefficient

- Standardized covariance is called the

  correlation coefficient or ρ

  ____

$$\rho_{(1,2)} = \frac{Cov(r_1, r_2)}{\sigma_1 \times \sigma_2}$$

- ρ is always in the range -1 to +1.

ρ and diversification in a 2 stock portfolio

- What does $-1 < \rho(1,2) < 1$ imply?
  - If $-1 < \rho(1,2) < 1$ then

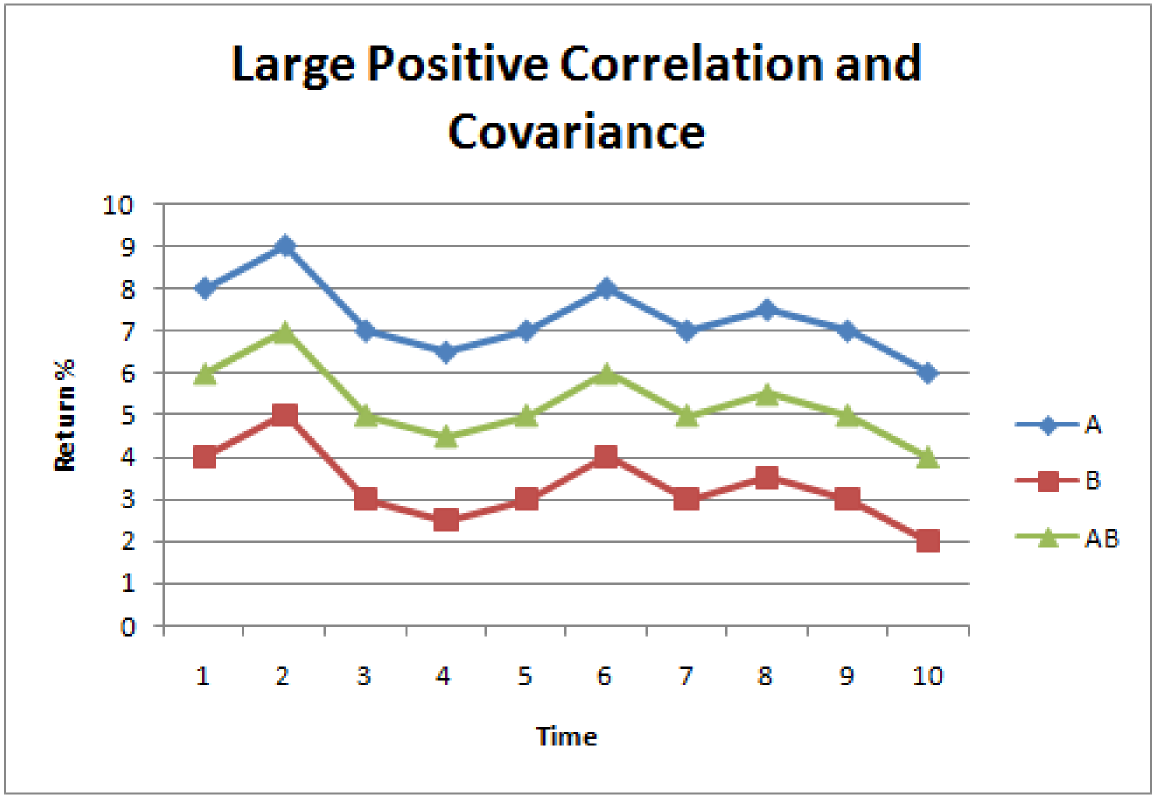
There are some diversification benefits from combining stocks 1 and 2 into a portfolio.

$\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2 \, Cov(r_1 r_2)$

And since $Cov(r_1 r_2) = \rho_{1,2}\sigma_1\sigma_2$

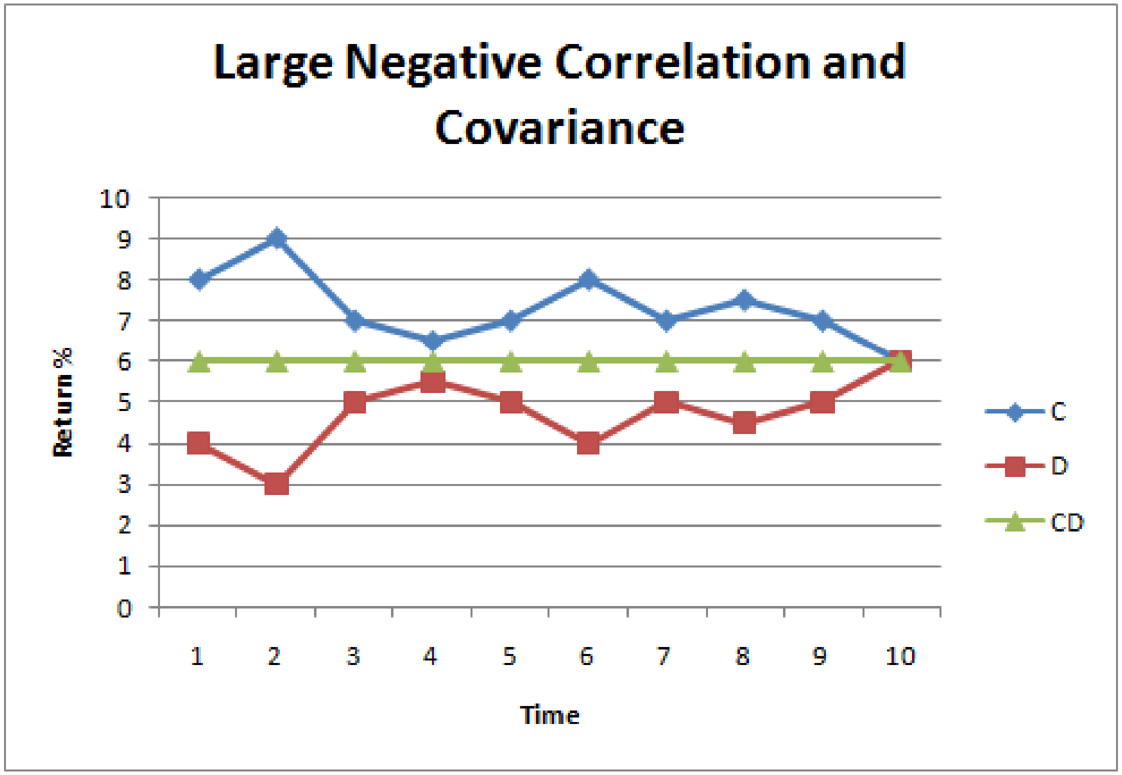
$\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2 \, \rho_{1,2}\sigma_1\sigma_2$

The effects of correlation & covariance on diversification



Asset A
Asset B
Portfolio AB

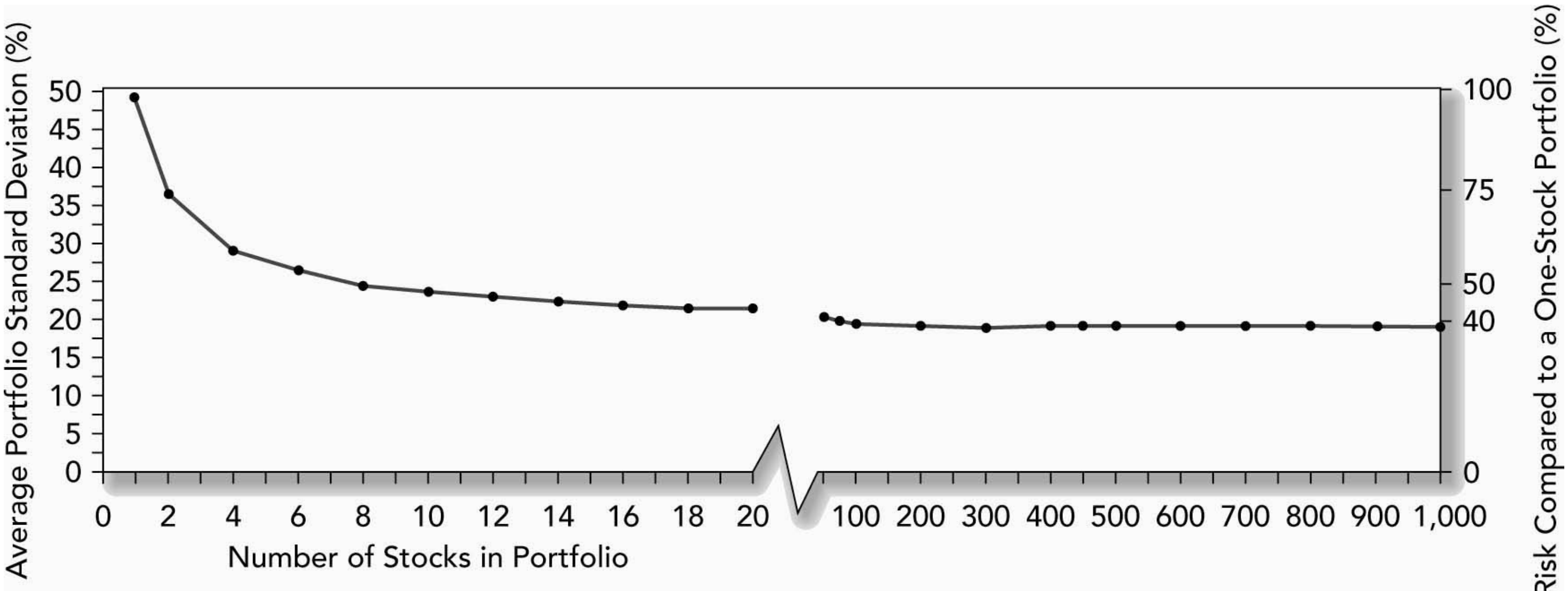
The effects of correlation & covariance on diversification



Asset C
Asset C
Portfolio CD

Naïve diversification

The power of diversification



Most of the diversifiable risk eliminated at 25 or so stocks

Two-Security Portfolio: Risk

$$\sigma_p^2 = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + 2W_1W_2\,Cov(r_1r_2)$$

Three-Security Portfolio n or Q = 3

$$\sigma^2_p = W_1^2\sigma_1^2 + W_2^2\sigma_2^2 + W_3^2\sigma_3^2$$

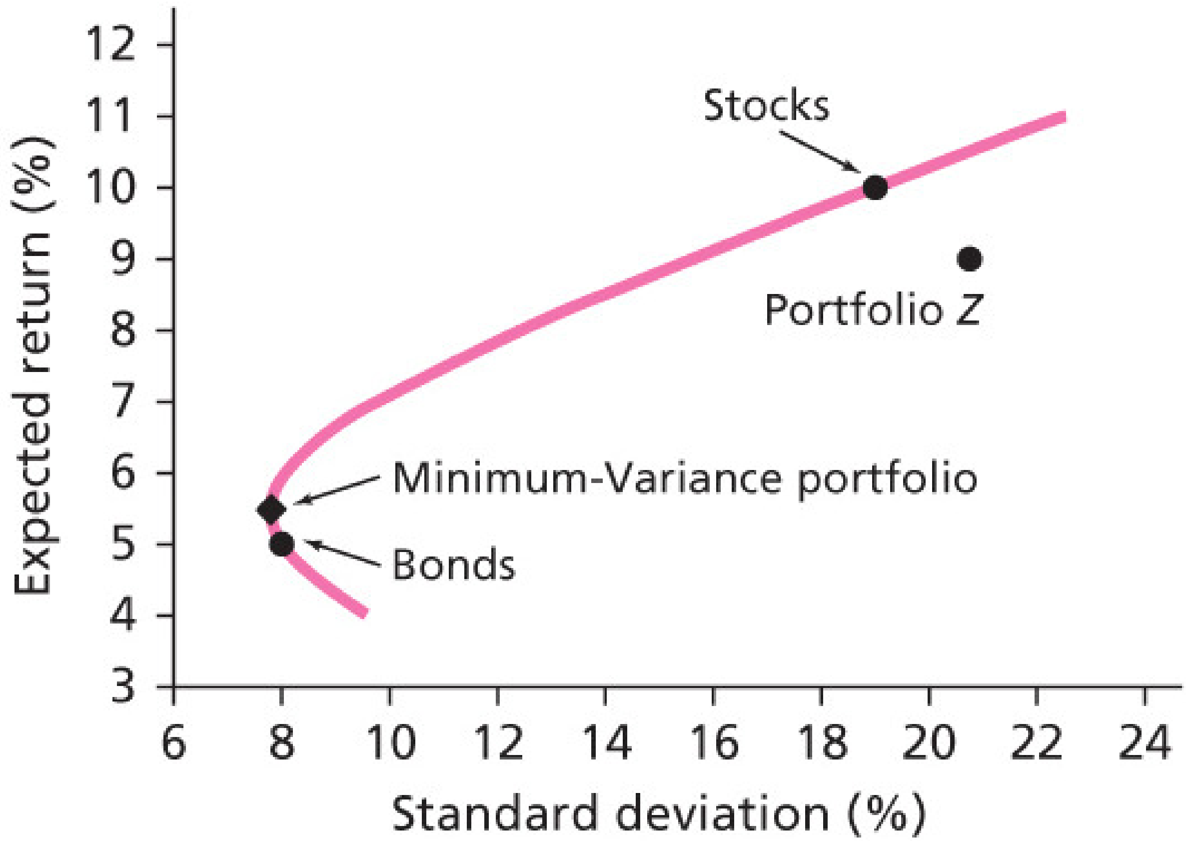
$$+ 2W_1W_2 \quad Cov(r_1r_2)$$

$$+ 2W_1W_3 \quad Cov(r_1r_3)$$

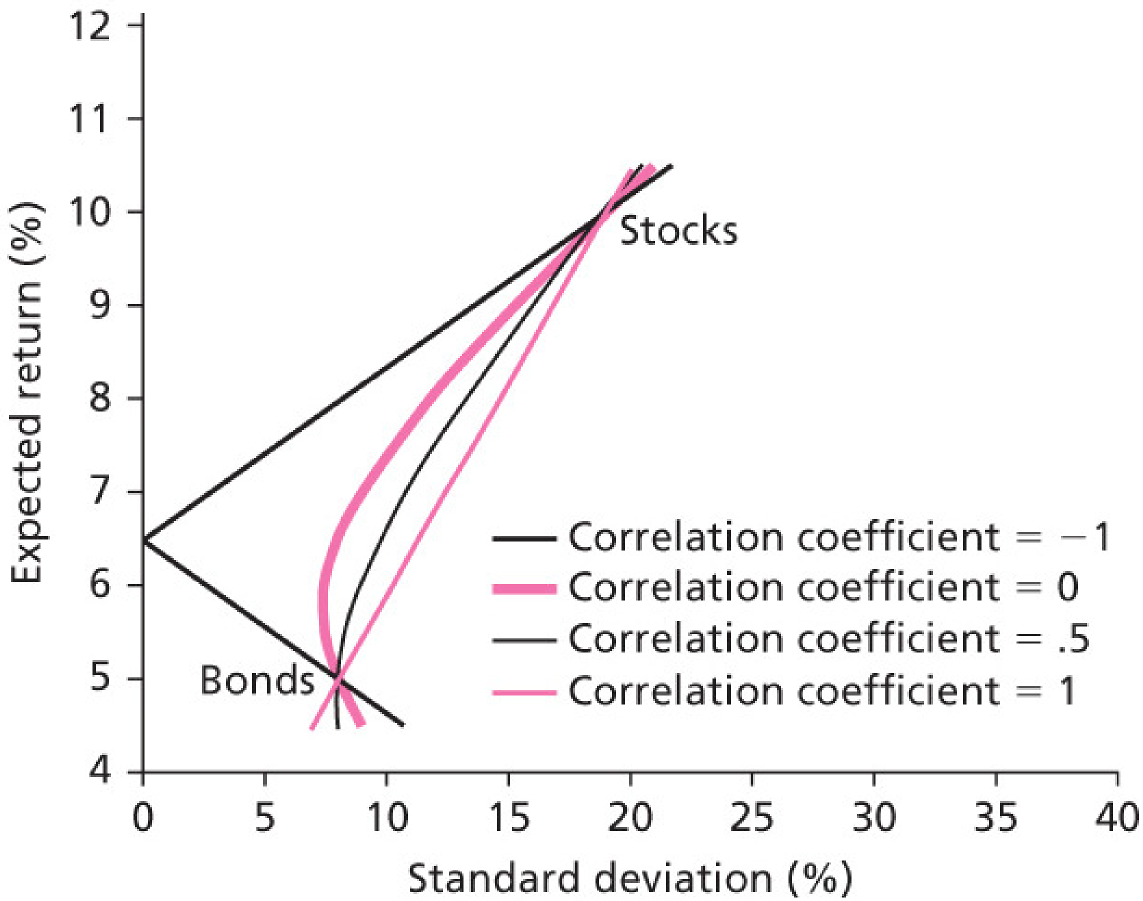
$$+ 2W_2W_3 \quad Cov(r_2r_3)$$

$$\sigma_p^2 = \sum_{I=1}^{Q}\sum_{J=1}^{Q}[W_I \, W_J \, Cov(r_I, r_J)]$$

Mean-Variance Criterion and Risky Portfolio Selection



Investment Opportunity Set for Stocks and Bonds with Various Correlations

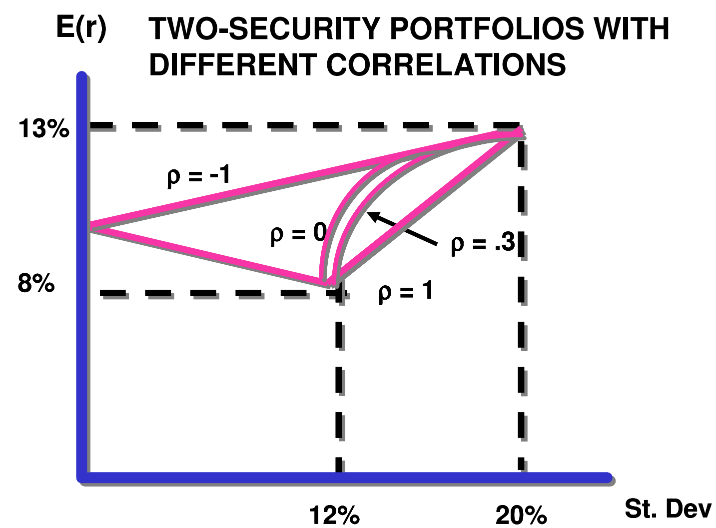


Minimum Variance Combinations $-1 < \rho < +1$

Choosing weights to minimize the portfolio variance

$$W_1 = \frac{\sigma_2^2 - Cov(r_1 r_2)}{\sigma_1^2 + \sigma_2^2 - 2Cov(r_1 r_2)}$$

$$W_2 = (1 - W_1)$$



Minimum Variance Combinations Example:

Stk 1	$E(r_1) = .10$	$\sigma_1 = .15$	$\rho_{12} = .2$
Stk 2	$E(r_2) = .14$	$\sigma_2 = .20$	

Solve for:

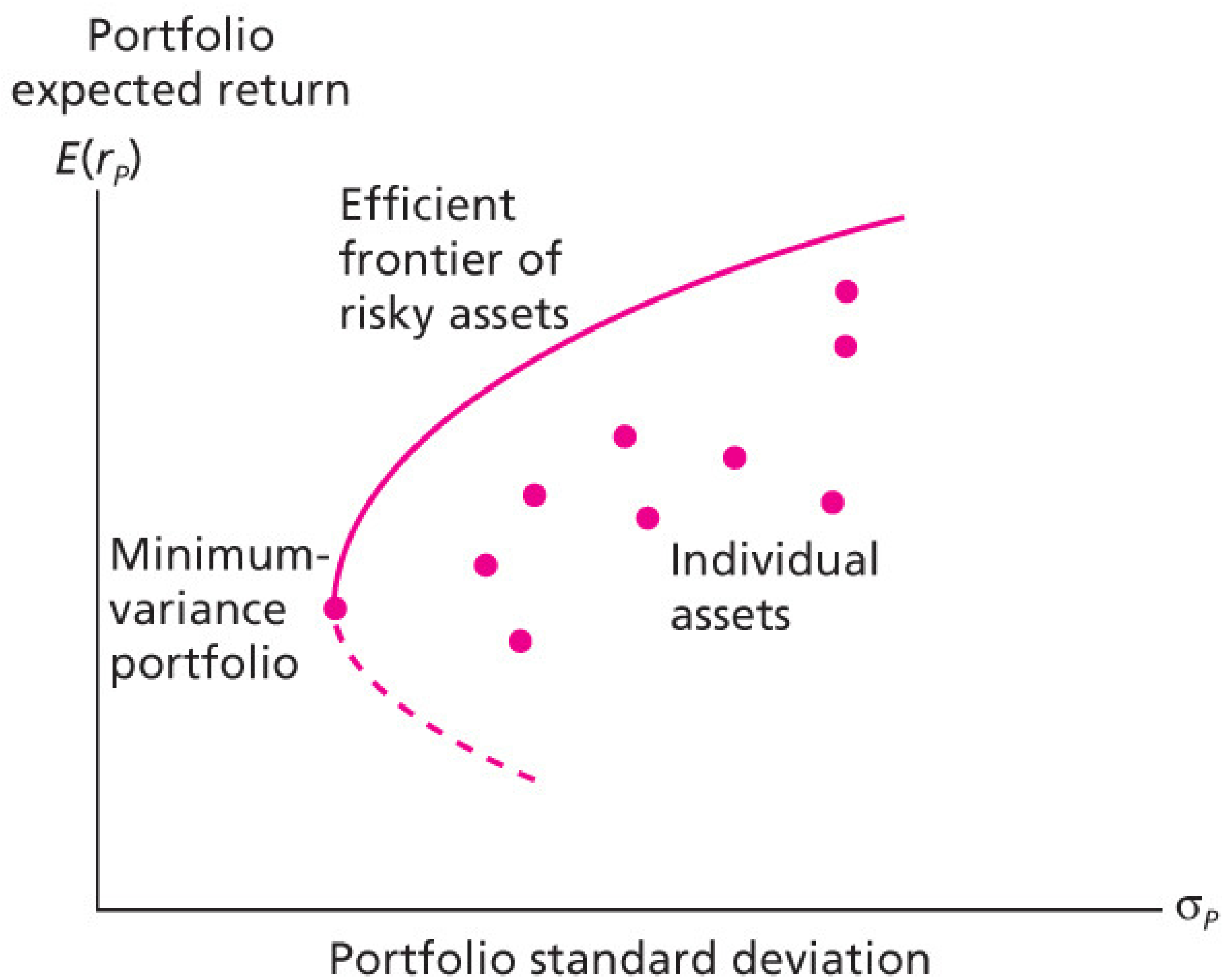
1. optimal weights
2. expected portfolio return
3. portfolio standard deviation

Extending Concepts to All Securities

Consider:

1. All possible combinations of securities,
2. All possible weights
3. Combinations that provide the least risk for a given level of return and graph the result.

Extending Concepts to All Securities



Extending Concepts to All Securities

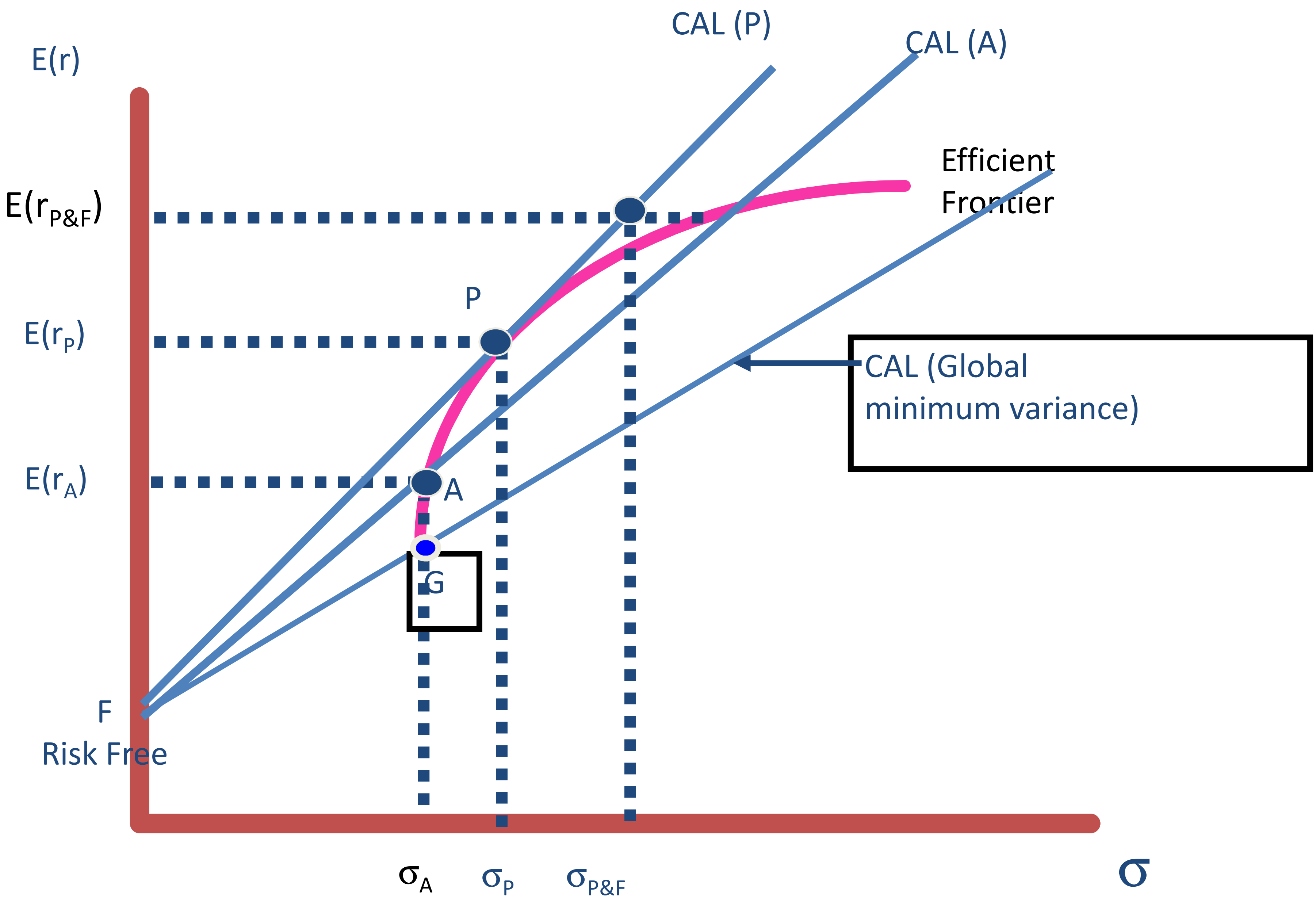
- The optimal combinations result in lowest level of risk for a given return
- The optimal trade-off is described as the efficient frontier
- These portfolios are dominant

Including Riskless Investments

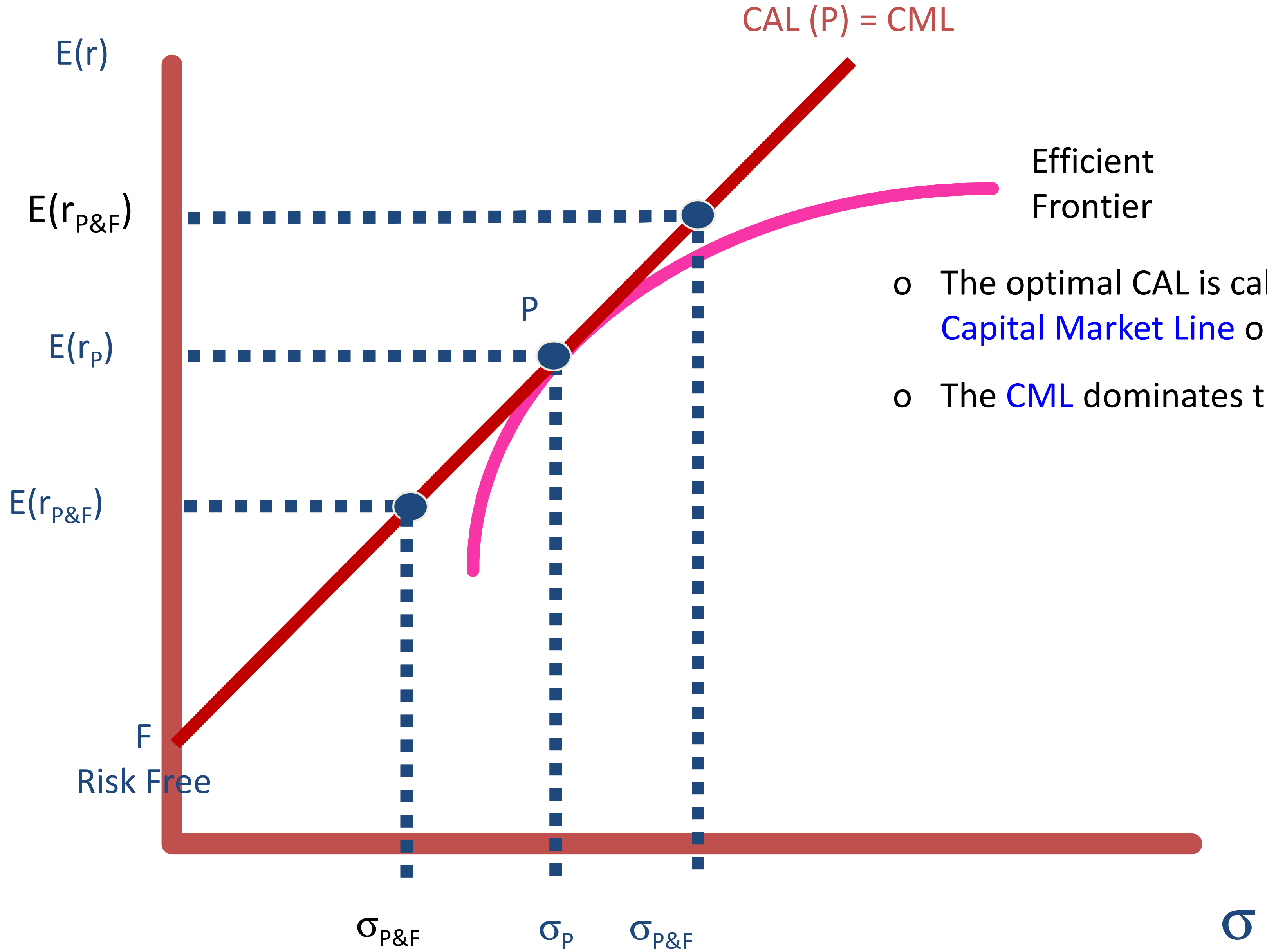
- The optimal combination becomes linear

- A single combination of risky and riskless assets will dominate

ALTERNATIVE CALS



The Capital Market Line or CML

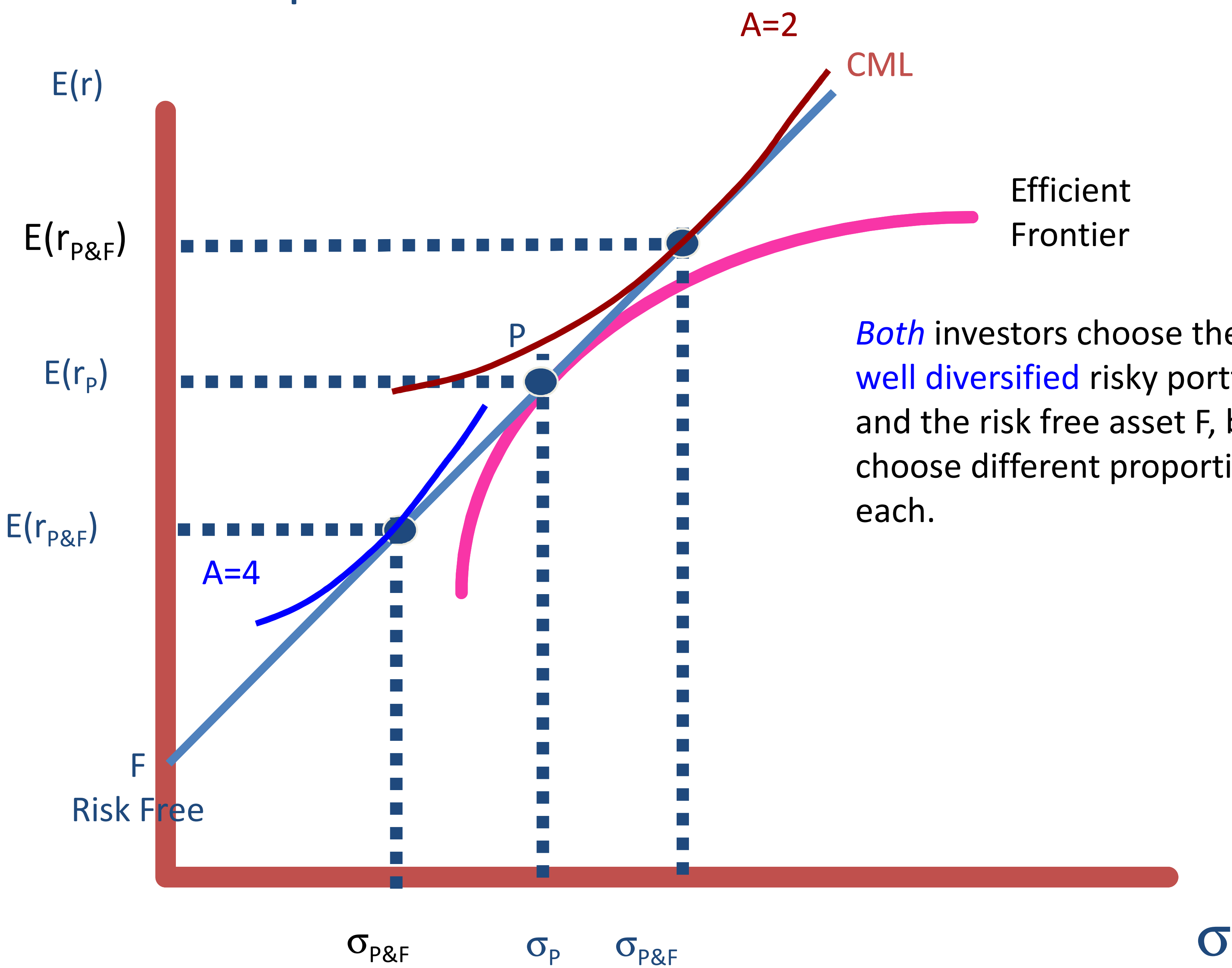


- The optimal CAL is called the Capital Market Line or CML
- The CML dominates the EF

Dominant CAL with a Risk-Free Investment (F)

- CAL(P) = Capital Market Line or CML dominates other lines because it has the the largest slope
- Slope = $(E(r_p) - rf) / \sigma_p$
  (CML maximizes the slope or the return per unit of risk or it equivalently maximizes the Sharpe ratio)
- Regardless of risk preferences some combinations of P & F dominate

The Capital Market Line or CML



Both investors choose the same well diversified risky portfolio P and the risk free asset F, but they choose different proportions of each.