Chapter 5

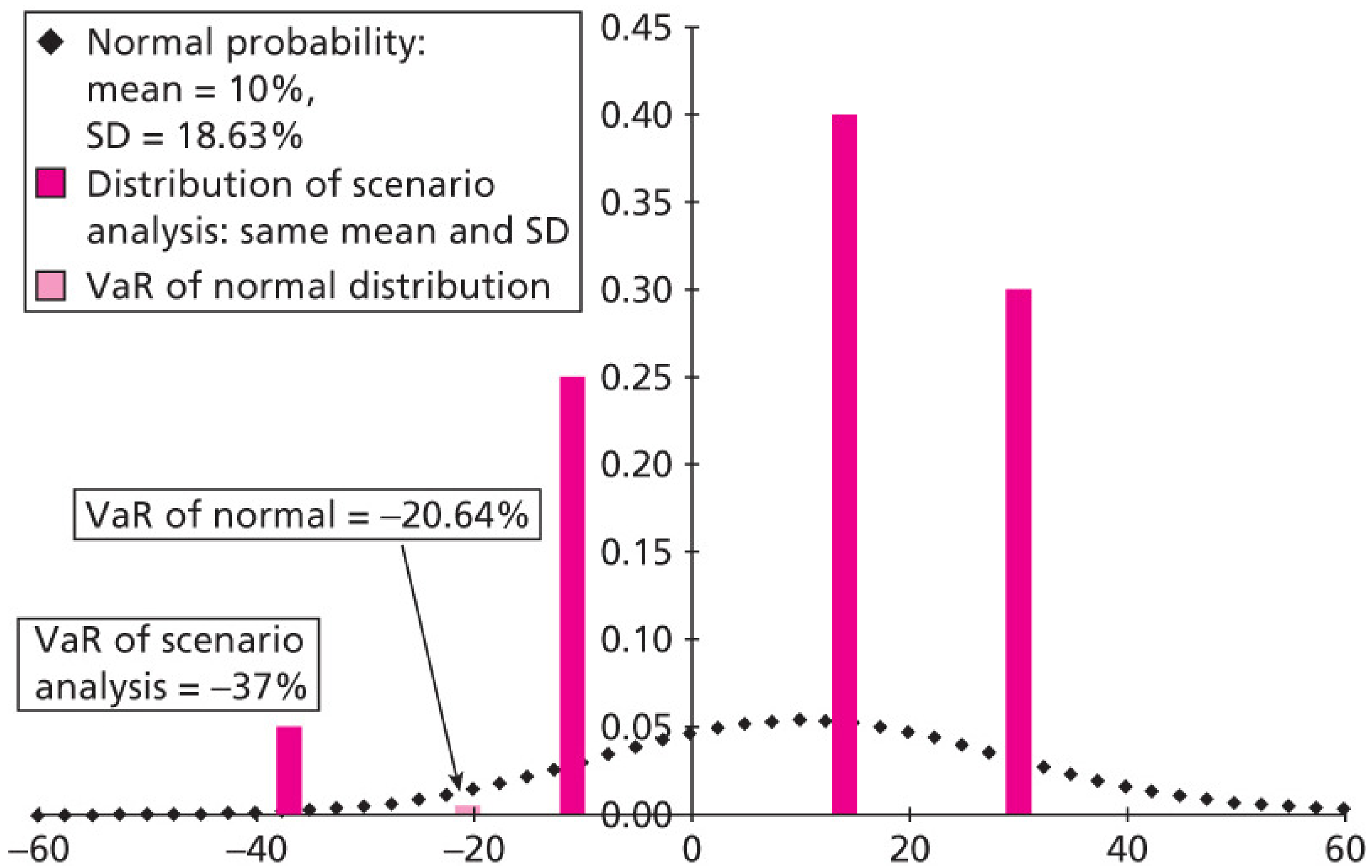
Risk and Return

(Cont'd)

Value at Risk (VaR)

			Column B x	Deviation from	Column B x
Scenario	Prob.	HPR (%)	Column C	Mean Return	Squared Deviation
1 Severe recession	0.05	-37	-1.85	-47.00	110.45
2 Mild recession	0.25	-11	-2.75	-21.00	110.25
3 Normal growth	0.40	14	5.60	4.00	6.40
4 Boom	0.30	30	9.00	20.00	120.00
		Expected return =	10.00	Variance =	347.10
				Std(%)=	18.63

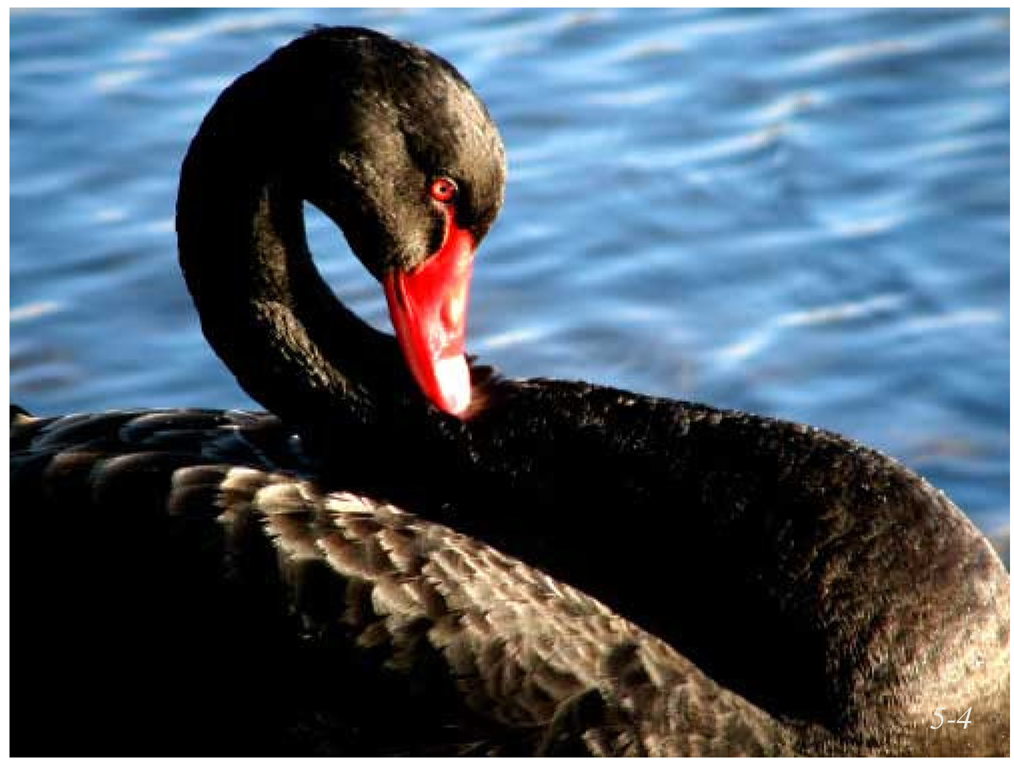
Value at Risk (VaR)



“Tail Risk”

Skew: measure of the asymmetry of a probability distribution

Kurtosis: measure of the fatness of the tails of a probability distribution

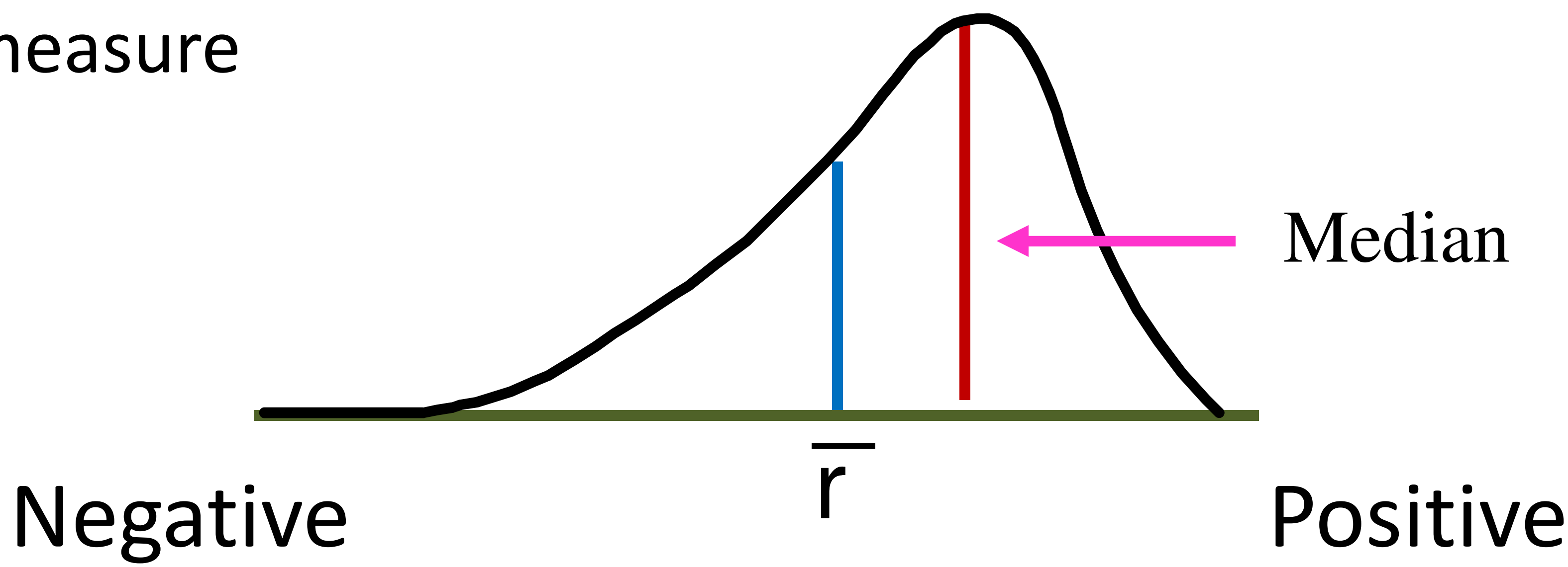


Skewed Distribution: Large Negative Returns Possible (Left Skewed)

Implication?

$\bar{r}$ *= average*

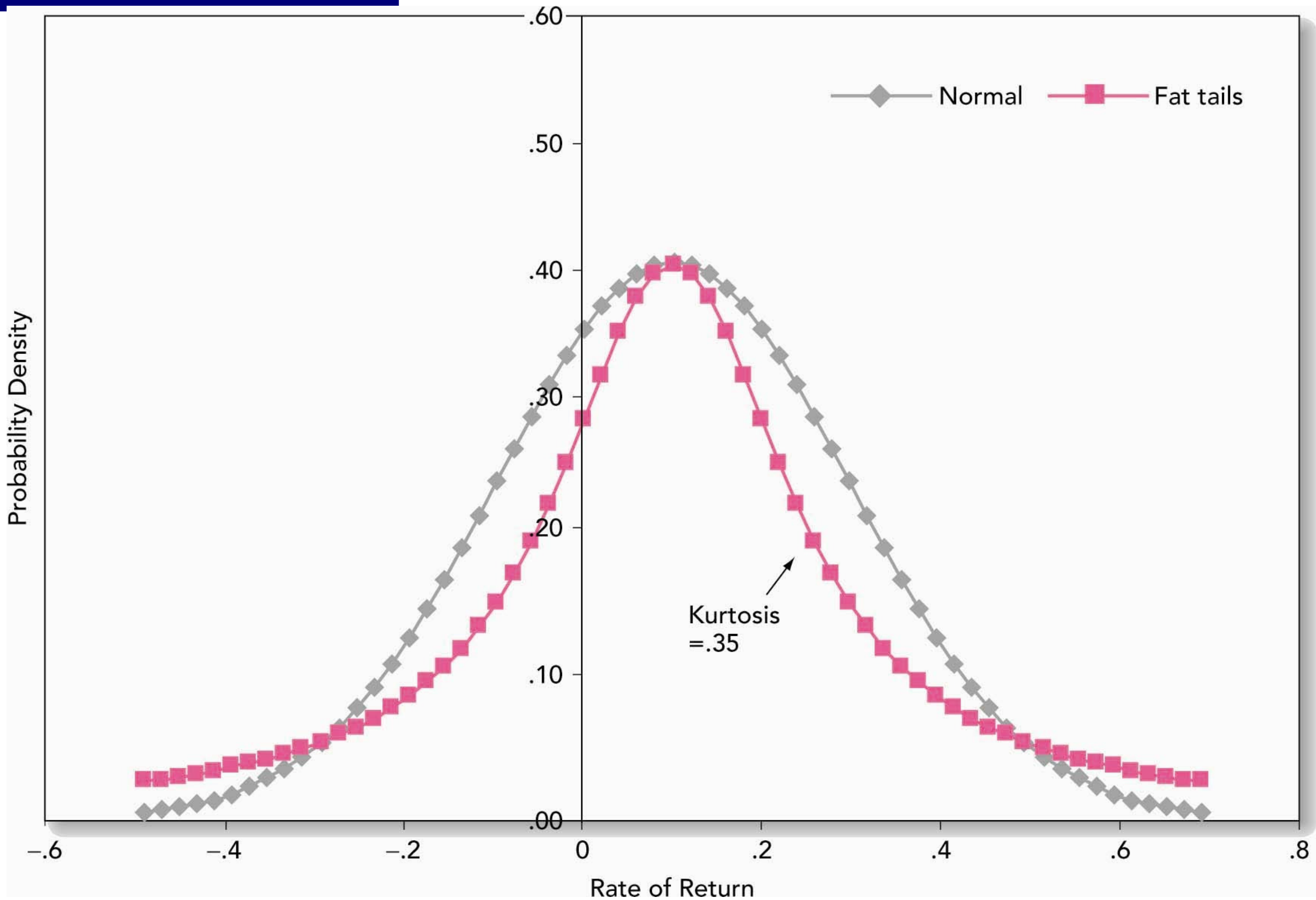
σ is an incomplete risk measure



Implication?

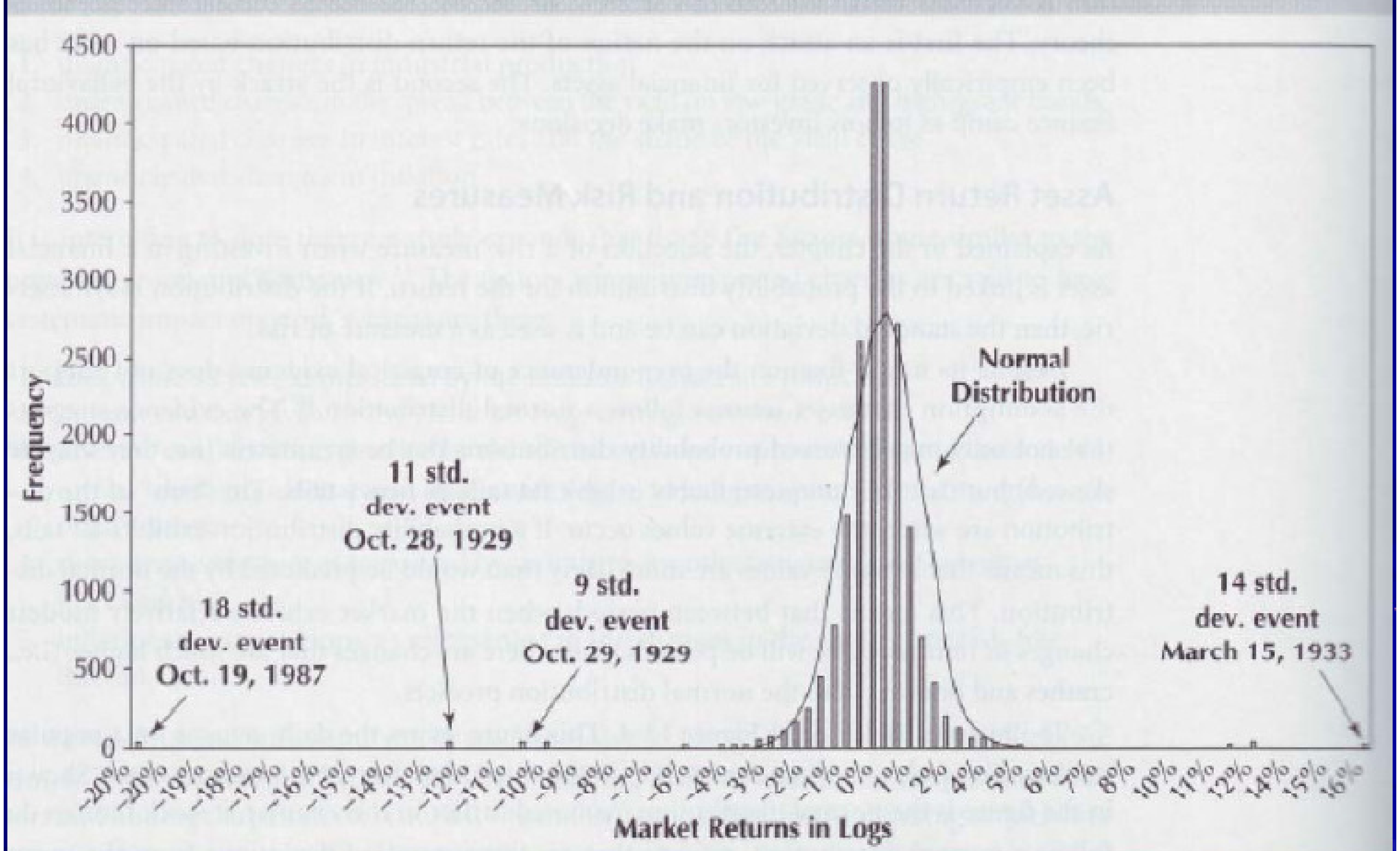
σ is an incomplete risk measure

Leptokurtosis



History

Figure 12-4 Distribution of Daily S&P 500 Returns (1928–April 2006)



Source: *Exhibit 4 in Jeremy Grantham, "Risk Management in Investing (Part Two): Risk and the Passage of Time," Letters to the Investment Committee VII (April 2006)*. © 2006 GMO LLC.

Annual Holding Period Returns Statistics 1926-2008

From Table 5.3

Series	Geom. Mean%	Arith. Mean%	Excess Return%	Kurt.	Skew.
World Stk	9.20	11.00	7.25	1.03	-0.16
US Lg. Stk	9.34	11.43	7.68	-0.10	-0.26
Sm. Stk	11.43	17.26	13.51	1.60	0.81
World Bnd	5.56	5.92	2.17	1.10	0.77
LT Bond	5.31	5.60	1.85	0.80	0.51

- *Geometric mean:*
  Best measure of compound historical return

- *Arithmetic Mean:*
  Expected return

- *Deviations from normality?*

Deviations from Normality: Another Measure

	Portfolio		
	World Stock	**US Small Stock**	**US Large Stock**
Arithmetic Average	**.1100**	**.1726**	**.1143**
Geometric Average	**.0920**	**.1143**	**.0934**
Difference	**.0180**	.0483	**.0209**
½ Historical Variance	**.0186**	.0694	**.0214**

If returns are normally distributed then:

Arithmetic Average – Geometric Average = ½ σ^2

The comparisons above indicate that US Small Stocks may have deviations from normality and therefore VaR may be an important risk measure for this class.

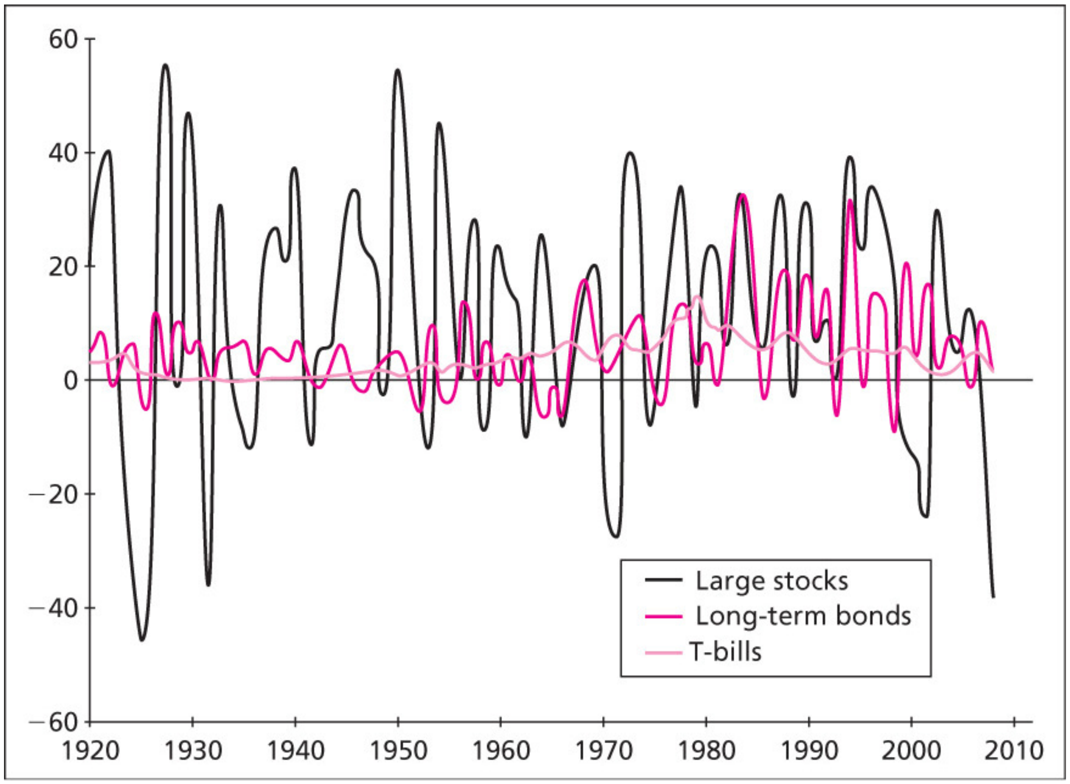
Risk Premium & Risk Aversion

- **The risk free rate is the rate of return that can be earned with certainty.**
- **The risk premium is the difference between the expected return of a risky asset and the risk-free rate.**

Excess Return or Risk Premium$_{asset}$ = $E[r_p] - r_f$

Risk aversion is an investor's reluctance to accept risk.

Rates of return on stocks, bonds and bills, 1926-2008



r_p

Sharpe Ratio:

$$Sharpe\ Ratio = \frac{Portfolio\ Risk\ Premium}{S.D.\ of\ Portfolio\ Excess\ Return}$$

$$= \frac{E(r_p) - r_f}{\sigma_p}$$

How is the aversion to accept risk overcome?

By offering investors a higher risk premium.

Historical Real Returns & Sharpe Ratios

Series	Real Returns%	Sharpe Ratio
World Stk	6.00	0.37
US Lg. Stk	6.13	0.37
Sm. Stk	8.17	0.36
World Bnd	2.46	0.24
LT Bond	2.22	0.24

Capital Allocation

- Possible to split funds between risky and risk free assets
- Risk free asset: T-Bills
- Risky asset: stock (or a portfolio of stocks)

Capital Allocation

- Examine risk/return tradeoff
- Demonstrate how different degrees of risk aversion affect allocation between risky and risk free assets
- Learn how to use leverage to achieve the desired risk/return profile

Allocating Capital Between Risky & Risk-Free Assets

- **Example. Your total wealth is $10,000. You put $2,500 in risk free T-Bills and $7,500 in a stock portfolio invested as follows:**

Risky Portfolio:

- **Stock A you put ______** $2,500
- **Stock B you put ______** $3,000
- **Stock C you put ______** $2,000

$7,500

Allocating Capital Between Risky & Risk-Free Assets

Stock A $2,500
Stock B $3,000
Stock C $2,000

Weights in rp

– W_A =	$2,500 / $7,500 =	33.33%
– W_B =	$3,000 / $7,500 =	40.00%
– W_C =	$2,000 / $7,500 =	<u>26.67%</u>
		100.00%

The complete portfolio includes the riskless investment and r_p.

Your total wealth is $10,000. You put $2,500 in risk free T-Bills and $7,500 in a stock portfolio invested as follows

W_{rf} = 25%; W_{rp} = 75%

In the <u>complete</u> portfolio

W_A = 0.75 x 33.33% = 25%;

W_B = 0.75 x 40.00% = 30%

W_C = 0.75 x 26.67% = 20%;

W_{rf} = 25%

Example

$r_f = 5\%$

$\sigma_{rf} = 0\%$

$E(r_p) = 14\%$

$\sigma_{rp} = 22\%$

$y = \%$ in r_p

$(1-y) = \%$ in rf

Expected Returns for Combinations

$$E(r_C) = yE(r_p) + (1 - y)r_f$$

$$\sigma_c = y\sigma_{rp} + (1\text{-}y)\sigma_{rf}$$

$E(r_C)$ = Return for complete or combined portfolio

$r_f = 5\%$	$\sigma_{rf} = 0\%$
$E(r_p) = 14\%$	$\sigma_{rp} = 22\%$
y = % in r_p	$(1\text{-}y)$ = % in rf

For example, let y = ____ 0.75

$E(r_C)$ = (.75 x .14) + (.25 x .05)
$E(r_C)$ = .1175 or 11.75%

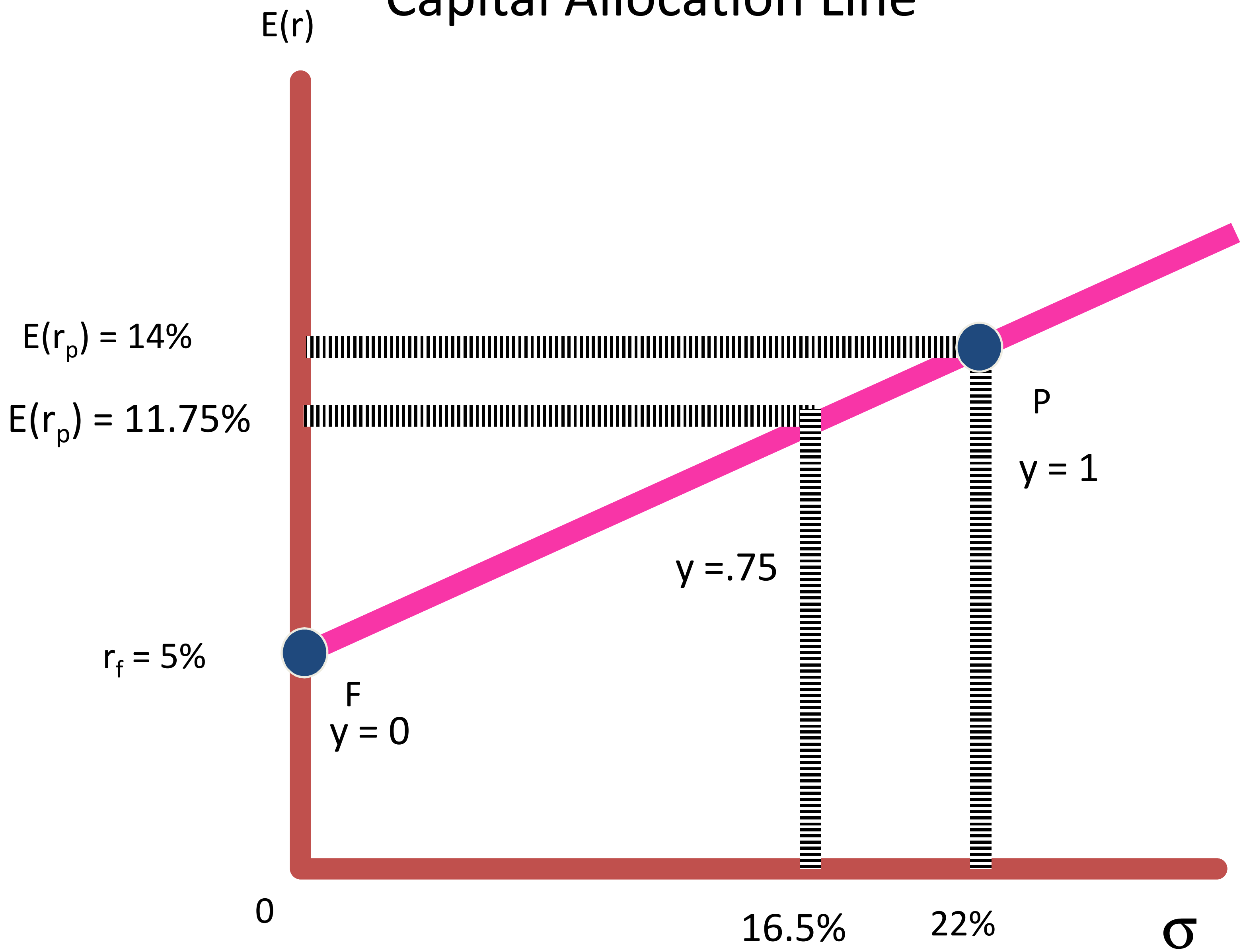
$\sigma_C = y\sigma_{rp} + (1\text{-}y)\sigma_{rf}$
σ_C = (0.75 x 0.22) + (0.25 x 0) = 0.165 or 16.5%

Complete portfolio

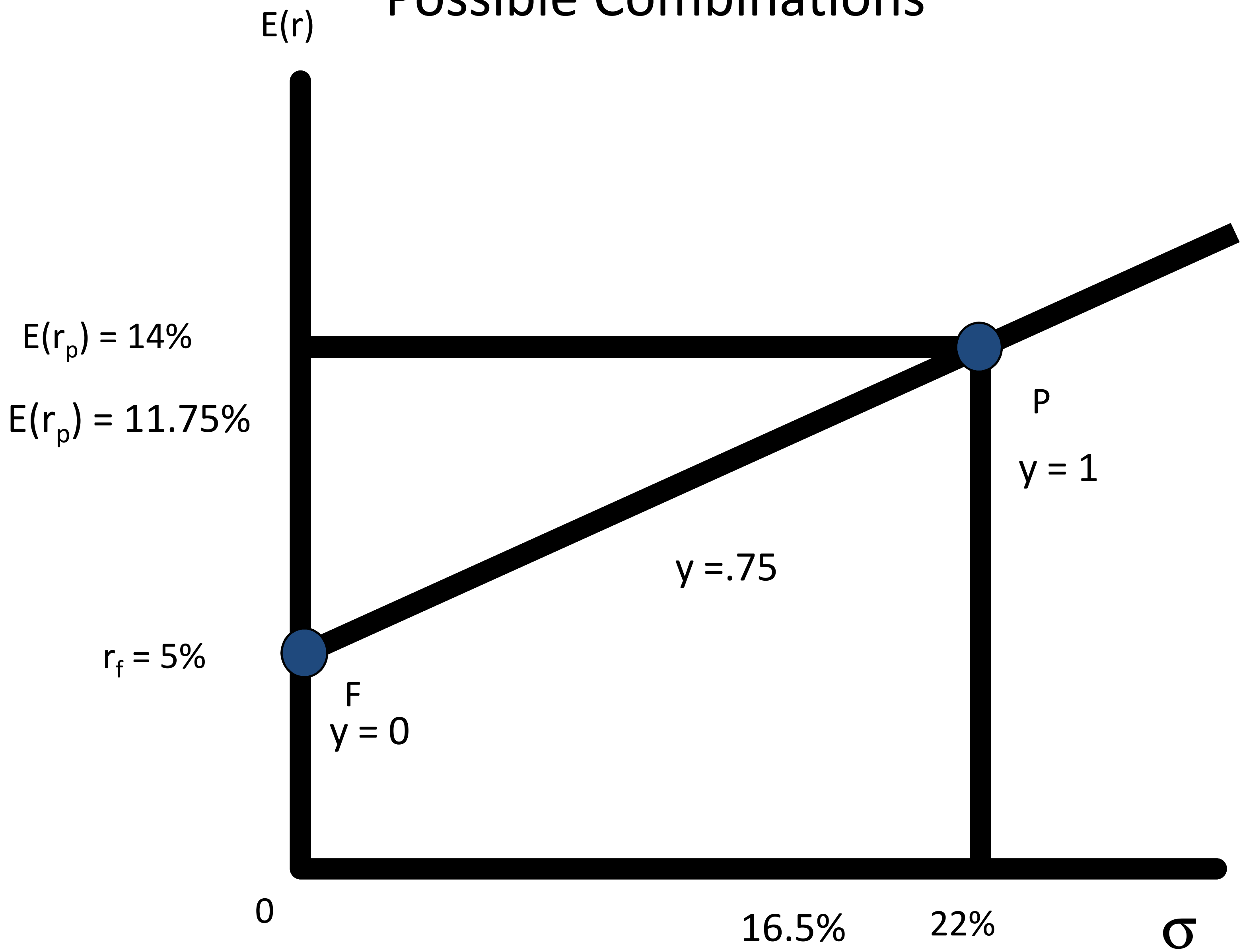
$$E(r_c) = yE(r_p) + (1 - y)rf$$

$$\sigma_c = y\sigma_{rp}$$

Capital Allocation Line



Possible Combinations



Combinations Without Leverage

$r_f = 5\%$ $\sigma_{rf} = 0\%$

$E(r_p) = 14\%$ $\sigma_{rp} = 22\%$

$y = \%$ in r_p $(1-y) = \%$ in rf

Since $\sigma_{rf} = 0$

$\sigma_c = y\ \sigma_p$

If y = .75, then

$\sigma_c =$ $75(.22) = 16.5\%$

$E(r_c) = yE(r_p) + (1 - y)rf$

$y = .75$

$E(r_c) =$ $(.75)(.14) + (.25)(.05) = 11.75\%$

If y = 1

$\sigma_c =$ $1(.22) = 22\%$

$y = 1$

$E(r_c) =$ $(1)(.14) + (0)(.05) = 14.00\%$

If y = 0

$\sigma_c =$ $0(.22) = 0\%$

$y = 0$

$E(r_c) =$ $(0)(.14) + (1)(.05) = 5.00\%$

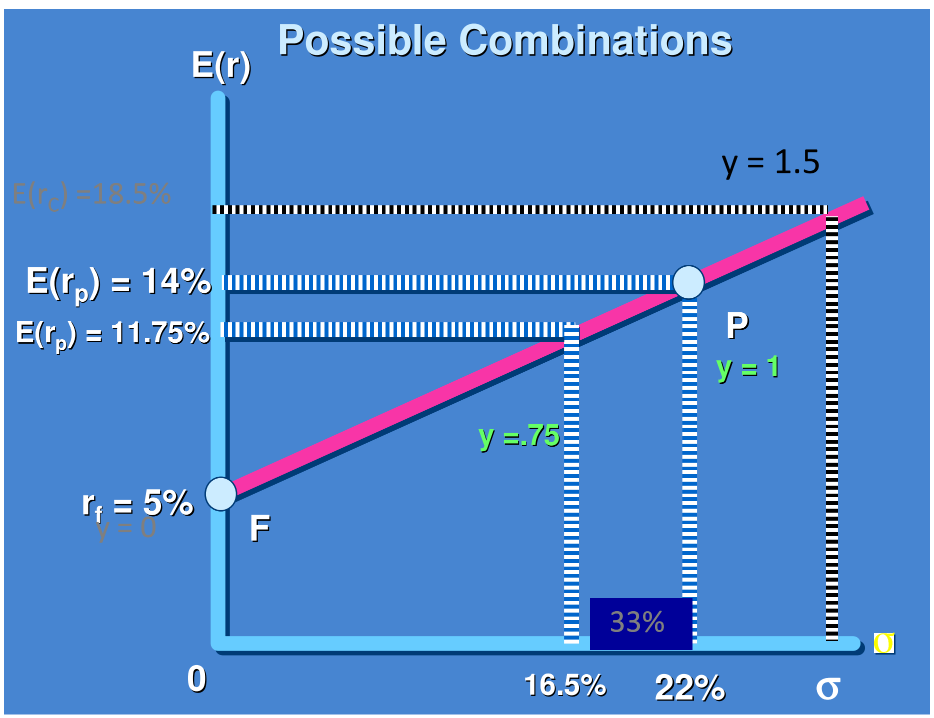
Using Leverage with Capital Allocation Line

Borrow at the Risk-Free Rate and invest in stock

Using 50% Leverage (y = 1.5).

$E(r_c)$ = (1.5) (.14) + (-.5) (.05) = 0.185 = 18.5%

σ_c = (1.5) (.22) = 0.33 or 33%

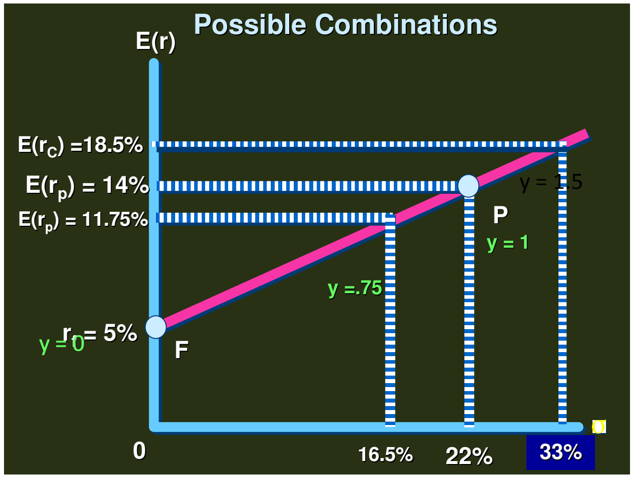


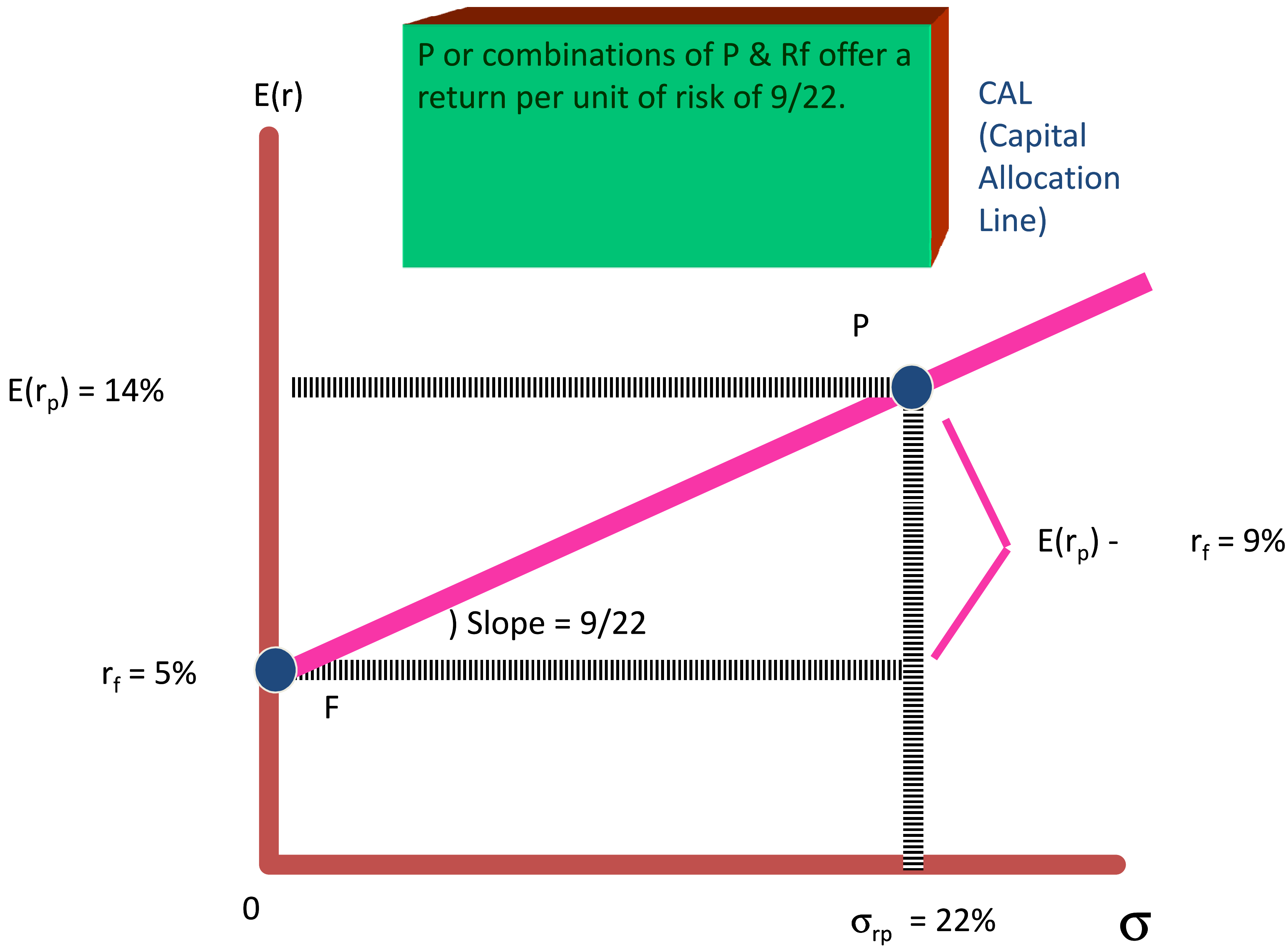
r_f = 5%	σ_{rf} = 0%
$E(r_p)$ = 14%	σ_{rp} = 22%
y = % in r_p	(1-y) = % in rf

Risk Aversion and Allocation

- **Greater levels of risk aversion lead investors to choose larger proportions of the risk free rate**

- Lower levels of risk aversion lead investors to choose larger proportions of the portfolio of risky assets
- Willingness to accept high levels of risk for high levels of returns would result in leveraged combinations





P or combinations of P & Rf offer a return per unit of risk of 9/22.
CAL (Capital Allocation Line)
E(r)
P
$E(r_p) = 14\%$
$E(r_p) - r_f = 9\%$
) Slope = 9/22
$r_f = 5\%$
F
0
$\sigma_{rp} = 22\%$
σ