Chapter 5

Risk and Return

(Cont'd)

Measuring Ex-Post (Past) Returns

Q: When should you use the GAR and when should you use the AAR?

A1: When you are evaluating PAST RESULTS (ex-post):

- Use the AAR (average without compounding) if you *ARE NOT* reinvesting any cash flows received before the end of the period.
- Use the GAR (average with compounding) if you *ARE* reinvesting any cash flows received before the end of the period.

A2: When you are trying to estimate an expected return (ex-ante return):

- Use the AAR

Expected Return

<u>Expected returns</u>

$$E(r) = \sum_{s} p(s) r(s)$$

$p(s)$ = probability of a state

$r(s)$ = return if a state occurs

s = state

Expected Return (Example)

State	Prob. of State	r in State
1	.1	-.05
2	.2	.05
3	.4	.15
4	.2	.25
5	.1	.35

$E(r) = (.1)(-.05) + (.2)(.05)\ldots + (.1)(.35)$

$E(r) = .15$

Variance or Dispersion of Returns

Variance:

Standard deviation = $[\text{variance}]^{1/2}$

Using Our Example:

$$\text{Var} = [(.1)(-.05-.15)^2+(.2)(.05-.15)^2 \ldots + .1(.35-.15)^2]$$

$$\text{Var} = .01199$$

$$\text{S.D.} = [.01199]^{1/2} = .1095$$

Numerical Example: Subjective or Scenario Distributions

State	Prob. of State	Return
1	.2	- .05
2	.5	.05
3	.3	.15

E(r) = (.2)(-0.05) + (.5)(0.05) + (.3)(0.15) = 6%

$$\sigma^2 = \sum_s p(s) \times [r_s - E(r)]^2$$

σ^2 = [(.2)(-0.05-0.06)^2 + (.5)(0.05- 0.06)^2 + (.3)(0.15-0.06)^2]

σ^2 = 0.0049%^{2}

σ = [0.0049]$^{1/2}$ = .07 or 7%

Expost Expected Return & σ

$$\bar{r} = \sum_{T=1}^{n} \frac{HPR_T}{n}$$

$\bar{r}$ = average HPR

n = # observations

$$\text{Expost Variance}: \sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2$$

$$\text{Expost Standard Deviation}: \sigma = \sqrt{\sigma^2}$$

Annualizing the statistics:

$$\bar{r}_{annual} = \bar{r}_{period} \times \#\,periods$$

$$\sigma_{annual} = \sigma_{period} \times \sqrt{\#\,periods}$$

Obs	DIS	$(r - r_{avg})^2$	Monthly HPRs, Source Yahoo finance
1	-0.035417	0.002212808	9/3/2002
2	0.093199	0.006654508	10/1/2002
3	0.15756	0.021297275	11/1/2002
4	-0.200637	0.045054632	12/2/2002
5	0.068249	0.00320644	1/2/2003
6	-0.026188	0.001429702	2/3/2003
7	-0.00183	0.000181016	3/3/2003
8	0.087924	0.005821766	4/1/2003
9	0.050211	0.001489002	5/1/2003
10	0.004734	4.74648E-05	6/2/2003
11	0.099052	0.00764371	7/1/2003
12	-0.068896	0.006483384	8/1/2003
13	-0.016478	0.000789704	9/2/2003
14	0.109174	0.009516098	10/1/2003
15	0.019343	5.95893E-05	11/3/2003
16	0.019409	6.06076E-05	12/1/2003
17	0.02829	0.000277753	1/2/2004
18	0.095035	0.00695741	2/2/2004
19	-0.061342	0.005324028	3/1/2004
20	-0.085344	0.00940277	4/1/2004
21	0.018851	5.22376E-05	5/3/2004
22	0.079128	0.004556811	6/1/2004
23	-0.103832	0.013330149	7/1/2004
24	-0.028414	0.001603051	8/2/2004
25	0.004562	4.98687E-05	9/1/2004
26	0.105671	0.008844901	10/1/2004
27	0.061998	0.002537528	11/1/2004
28	0.041453	0.000889761	12/1/2004
29	0.028856	0.000296963	1/3/2005
30	-0.024453	0.001301505	2/1/2005

Obs	DIS	$(r - r_{avg})^2$	Monthly HPRs, Source Yahoo finance
31	0.027334	0.000246811	3/1/2005
32	-0.088065	0.009937839	4/1/2005
33	0.037904	0.000690654	5/2/2005
34	-0.089915	0.010310121	6/1/2005
35	0.0179	3.93874E-05	7/1/2005
36	-0.017814	0.000866572	8/1/2005
37	-0.043956	0.003089121	9/1/2005
38	0.010042	2.50266E-06	10/3/2005
39	0.022495	0.00011818	11/1/2005
40	-0.029474	0.001689005	12/1/2005
41	0.05303	0.001714497	1/3/2006
42	0.09589	0.007100858	2/1/2006
43	-0.003618	0.000232311	3/1/2006
44	0.002526	8.27674E-05	4/3/2006
45	0.083361	0.005146208	5/1/2006
46	-0.016818	0.000808939	6/1/2006
47	-0.010537	0.000491104	7/3/2006
48	-0.001361	0.000168618	8/1/2006
49	0.04081	0.000851813	9/1/2006
50	0.01764	3.61885E-05	10/2/2006
51	0.047939	0.001318787	11/1/2006
52	0.044354	0.001071242	12/1/2006
53	0.02559	0.000195054	1/3/2007
54	-0.026861	0.001481106	2/1/2007
55	0.005228	4.09065E-05	3/1/2007
56	0.015723	1.68055E-05	4/2/2007
57	0.01298	1.83836E-06	5/1/2007
58	-0.038079	0.002470321	6/1/2007
59	-0.034545	0.002131602	7/2/2007
60	0.017857	0.000038854	8/1/2007

Average	**0.011624**		**0.219762458**
Variance	**0.003725**	$\Sigma (r - r_{avg})^2 =$	
Stdev	**0.061031**	**n**	**60**
		n-1	**59**
Annualized			
Average	**0.139486**		
Variance	**0.044697**		
Stdev	**0.211418**		

$$\bar{r} = \sum_{T=1}^{n} \frac{HPR_T}{n}$$

$\bar{r}$ = average HPR

n = # observations

$$\text{Expost Variance}: \sigma^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2$$

$$\text{Expost Standard Deviation}: \sigma = \sqrt{\sigma^2}$$

Annualizing the statistics:

$$\bar{r}_{annual} = \bar{r}_{monthly} \times 12$$

$$\sigma_{annual} = \sigma_{monthly} \times \sqrt{12}$$

Using Ex-Post Returns to estimate Expected HPR

Estimating Expected HPR (E[r]) from ex-post data.

Use the arithmetic average of past returns as a forecast of expected future returns as we did and,

Perhaps apply some (usually ad-hoc) adjustment to past returns

Problems?

- Which historical time period?
- Have to adjust for current economic situation
    - Unstable averages
    - Stable risk

Characteristics of Probability Distributions

1. Mean: Arithmetic average & usually most likely ______________________________ _
2. Median: Middle observation _______________
3. Variance or standard deviation:

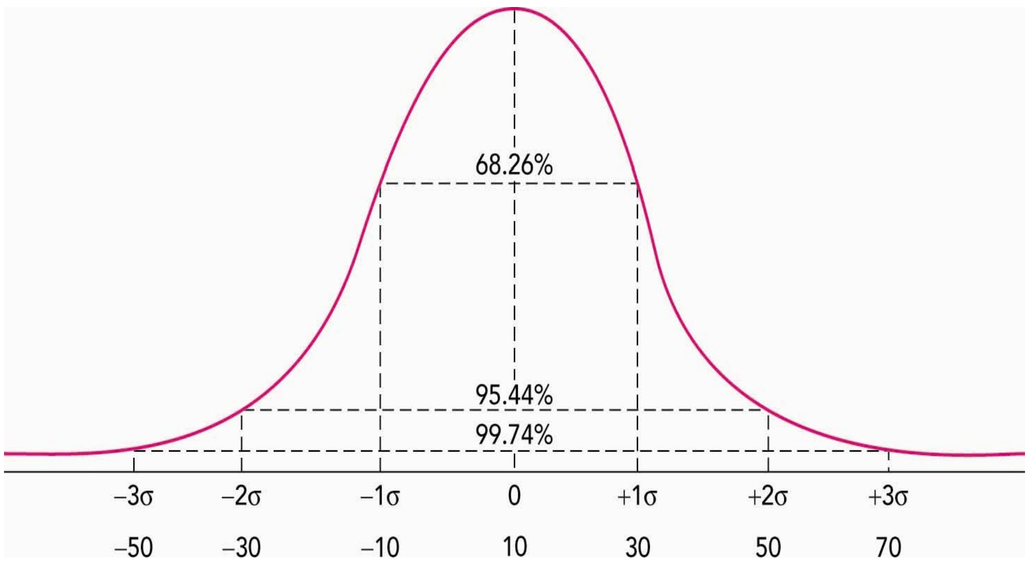
   Dispersion of returns about the mean
4. Skewness: Long tailed distribution, either side ___________________________
5. Leptokurtosis: Too many observations in the tails __________________________

- If a distribution is approximately normal, the distribution is fully described by Characteristics 1 and 3 _________________

Normal Distribution

Risk is the possibility of getting returns different from expected.

"68-95-99" Rule



E[r] = 10%

σ = 20%

Average = Median

Value at Risk (VaR)

Value at Risk attempts to answer the following question:

- **How many dollars can I expect to lose on my portfolio in a given time period at a given level of probability?**
- **The typical probability used is 5%.**
- **We need to know what HPR corresponds to a 5% probability.**
- **If returns are normally distributed then we can use a standard normal table or Excel to determine how many standard deviations below the mean represents a 5% probability:**
  - **From Excel: =Norminv (0.05,0,1) = -1.64485 standard deviations**

Value at Risk (VaR)

From the standard deviation we can find the corresponding level of the portfolio return:

$$VaR = E[r] + -1.64485\sigma$$

For Example:

A $500,000 stock portfolio has an annual expected return of 12% and a standard deviation of 35%. What is the portfolio VaR at a 5% probability level?

VaR = 0.12 + (-1.64485 * 0.35)

VaR = -45.57% (rounded slightly)

VaR$ = $500,000 x -.4557 = -$227,850

What does this number mean?

Value at Risk (VaR)

VaR versus standard deviation:

- **For normally distributed returns VaR is equivalent to standard deviation**

- **VaR adds value as a risk measure when return distributions are not normally distributed.**
  - **Actual 5% probability level will differ from 1.68445 standard deviations from the mean due to kurtosis and skewness.**

Value at Risk (VaR)

Scenario	Prob.	HPR (%)	Column B x Column C	Deviation from Mean Return	Column B x Squared Deviation
1 Severe recession	0.05	-37	-1.85	-47.00	110.45
2 Mild recession	0.25	-11	-2.75	-21.00	110.25
3 Normal growth	0.40	14	5.60	4.00	6.40
4 Boom	0.30	30	9.00	20.00	120.00
		Expected return =	10.00	Variance =	347.10
				Std(%)=	18.63

Value at Risk (VaR)